



A Method of Efficient Inclination Changes for Low-Thrust Spacecraft

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ABSTRACT

The evolution of low-thrust propulsion technologies has reached a point where such systems have become an economical option for many space missions. The development of efficient, low trip time control laws has received an increasing amount of attention in recent years, though few studies have examined the subject of inclination changing maneuvers in detail. A method for performing economical inclination changes through the use of an efficiency factor is derived from Lagrange's planetary equations. The efficiency factor can be used to regulate propellant expenditure at the expense of trip time. Such a method can be used for discontinuous-thrust transfers that offer reduced propellant masses and trip-times in comparison to continuous thrust transfers, while utilizing thrusters that operate at a lower specific impulse. Performance comparisons of transfers utilizing this approach with continuous-thrust transfers are generated through trajectory simulation and are presented in this paper.

NOMENCLATURE

a	semi-major axis
e	eccentricity
i	inclination
ω	argument of periapsis
v	true anomaly
I_{sp}	specific impulse
p	spacecraft propulsive power
η_p	spacecraft power subsystem efficiency
m_0	initial spacecraft mass
m_p	spacecraft propellant mass expenditure
MET	mission elapsed time
t	time
r	radius
n	mean motion
W	magnitude of orbit-normal thrust vector
$\eta_{\Delta i}$	efficiency of inclination change in given orbit
χ	minimum magnitude of $\eta_{\Delta i}$ to enable thrusting

INTRODUCTION

The evolution of low-thrust propulsion technologies has reached a point where such systems have become an economical option for many space mission applications. The development of efficient control laws has received an increasing amount of attention in recent years, though few studies have examined the subject of inclination changing maneuvers in detail.

The topics covered in this paper are the results of trajectory design work in which large changes in inclination were required for orbits of moderate eccentricity. The resulting methods allow the tradeoff between trip time and propellant mass to be quantified, thus giving a mission designer a convenient way of managing mission performance. There is a wide array of applications, including transfers between low-Earth orbit, the Moon, and other planets, where a significant change in inclination is likely to be required at some point in the mission.

Using Lagrange's planetary equations as a starting point, an efficiency factor can be derived that normalizes the time-rate of change in inclination at a given point in the orbit by the maximum value of di/dt in the orbit. Previous work along these lines has been conducted for in-plane orbital transfers utilizing discontinuous-thrust¹ and for continuous-thrust orbit raising maneuvers incorporating inclination changes². Methods for managing inclination through the efficiency factor for both continuous-thrust and discontinuous-thrust cases will be explained in the following sections of this paper.

BEHAVIOR OF di/dt

From Lagrange's planetary equations, the time-rate of change in inclination can be written³:

$$\frac{di}{dt} = \frac{Wr \cos(\omega + v)}{na^2 \sqrt{1 - e^2}} \quad (1)$$

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Using the expression for radius in Keplerian elements, Equation 1 can be written

$$\frac{di}{dt} = \frac{W\sqrt{1-e^2} \cos(\omega+v)}{na(1+e \cos(v))} \quad (2)$$

Plots of the magnitude of di/dt , from Equation 2, with respect to true anomaly and eccentricity are shown in Figures 1 and 2. A value of 1.0 is used for W , n , and a .

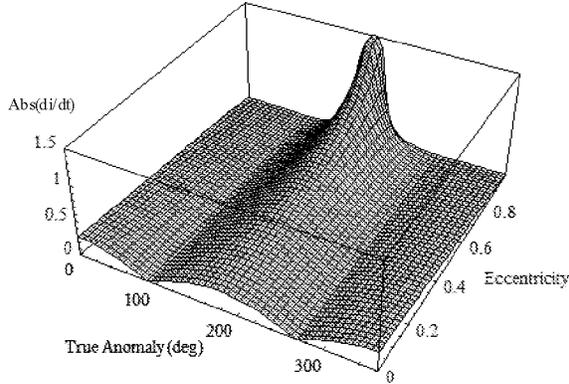


Figure 1: Abs(di/dt) vs. True Anomaly And Eccentricity, $\omega=0^\circ$

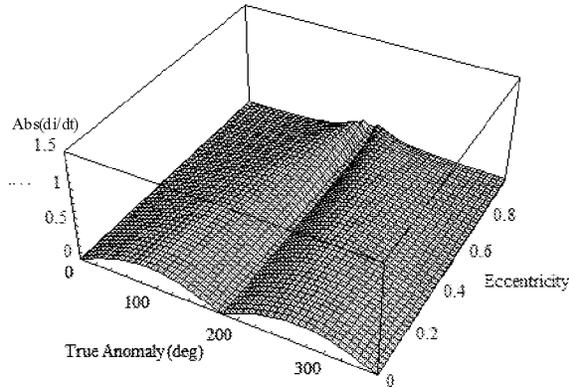


Figure 2: Abs(di/dt) vs. True Anomaly and Eccentricity, $\omega=0^\circ$

Note that the zeros always occur on the antinodal lines and the behavior is sinusoidal at an eccentricity of zero. As eccentricity is increased the behavior deviates from the sinusoidal case due to the presence of the radius term in the numerator of Equation 1. This suggests that sinusoidal inclination control laws developed in the past are not well suited for orbits of significant eccentricity.

CONTROL LAW APPLICATIONS

Two control-laws are considered here. The first is a continuous-thrust case in which orbit-normal thrust is scaled by the magnitude of di/dt . Excess thrust is diverted to the in-plane components of thrust to control semi-major axis or eccentricity as the mission requires. The second control law completely suspends orbit-normal thrust when di/dt is arbitrarily deemed to be unreasonably low. In this case the I_{sp} may be lowered, if possible, to reduce the time required to perform the inclination change. The result is a maneuver requiring less propellant mass to achieve a trip time equal to that of the continuous-thrust case, as will be demonstrated later in this paper.

Continuous-Thrust Control Law Derivation

The effectiveness of performing a change in inclination at a given point in an orbit may be gauged by comparing the value of di/dt per unit of orbit-normal thrust at a given point in the orbit with the maximum value of di/dt as a function of true anomaly. The resulting efficiency factor ($\eta_{\Delta i}$) may be used to scale the out of plane thrust.

The first step is to determine the extrema of Equation 2. The sign of di/dt is ignored as only the magnitude of the extrema is of interest. Thrust is assumed to be considerably low such that all elements aside from true anomaly are constant through a single orbital period. Equation 2 can be effectively simplified by removing all terms that are assumed to be constant, giving equation 3:

$$\frac{di}{dt} = \frac{\cos \omega \cdot \cos v - \sin \omega \cdot \sin v}{(1 + e \cdot \cos v)} \cdot Const \quad (3)$$

Taking the derivative of Equation 2 with respect to v and solving for the zeros yields the extrema:

$$v = -\sin^{-1}(e \sin(\omega)) - \omega \quad (4)$$

$$v = \sin^{-1}(e \sin(\omega)) - \omega + 180^\circ \quad (5)$$

The absolute maximum magnitude of di/dt can then be determined by comparing the results when these values are used in Equation 3. Now that di/dt_{max} is known, the ratio of di/dt at the current true anomaly to di/dt_{max} can be used to quantify the efficiency of orbit-normal thrusting in changing the inclination of an orbit. The constant terms will cancel, and can therefore be neglected. The efficiency of orbit normal thrusting to achieve an inclination change can then be defined as follows:

$$\eta_{\Delta i} = \frac{di/dt}{di/dt_{\max}} = \frac{\cos \omega \cos v - \sin \omega \sin v}{di/dt_{\max} (1 + e \cos v)} \quad (6)$$

Figures 3 through 5 show values of $\eta_{\Delta i}$ mapped onto various orbits. Note that the magnitude of di/dt is much larger near apoapsis for an orbit of significant eccentricity.

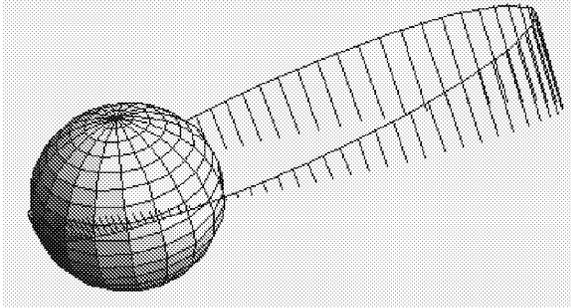


Figure 3: Orbit-Normal Thrust Vectors and Efficiency of Inclination Change, $\omega=0^\circ$, $e=0.5$

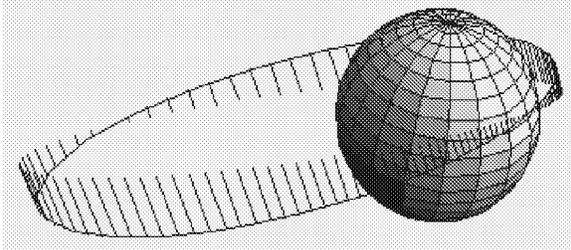


Figure 4: Orbit-Normal Thrust Vectors and Efficiency of Inclination Change, $\omega=90^\circ$, $e=0.5$

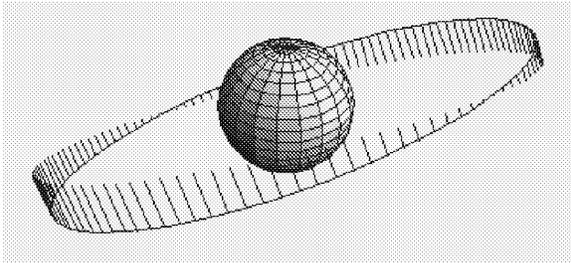


Figure 5: Orbit-Normal Thrust Vectors and Efficiency of Inclination Change, $\omega=0^\circ$, $e=0.0$

Discontinuous-Thrust Control Law Derivation

As has previously been demonstrated for in-plane control laws by Gefert and Hack¹, lower trip times and propellant mass expenditures may be achieved with thrusters of lower I_{sp} by coasting during portions of the

orbit where thrust has relatively little effect. In the case of an inclination change, $\eta_{\Delta i}$ can be used to quantify the effectiveness of orbit-normal thrust in changing a spacecraft's inclination at any given point.

Orbit-normal thrusting will be suspended when $\eta_{\Delta i}$ drops below some threshold value, henceforth referred to as χ . In the extreme cases, $\chi=1.0$ will result in infinitesimally short bursts of thrust at the true anomaly of di/dt_{\max} , given by Equation 2 and/or Equation 3. The other extreme, $\chi=0.0$, will result in continuous orbit-normal thrusting. The value of χ that will provide the desired tradeoff between trip time and propellant expenditure will, of course, vary from mission to mission. Figure 6, below, illustrates regions of thrusting and coasting for an eccentric orbit.

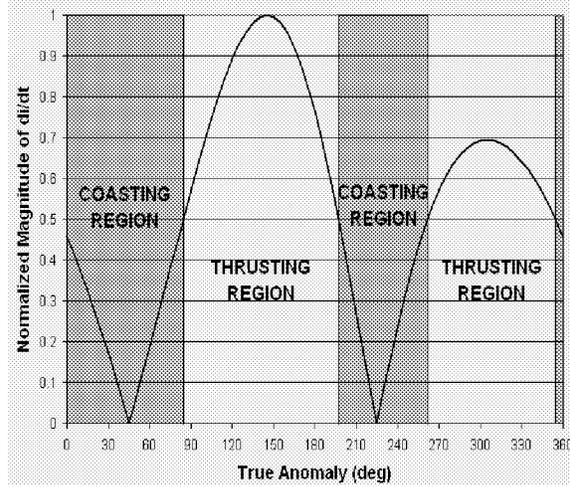


Figure 6: Thrusting and Coasting Regions for $e=0.25$, $\omega = 45^\circ$, and $\chi=0.5$

For this case the values of true anomaly at which $\eta_{\Delta i}$ rises above or drops below χ are desired, so that thrusting can be suspended and resumed at the appropriate points in the orbit. It is assumed that di/dt_{\max} is known from Equations 3 through 5. The values of true anomaly which satisfy the following equation are sought:

$$\eta_{\Delta i} = \frac{di/dt}{di/dt_{\max}} = \chi \quad (7)$$

Using the reduced definition of di/dt given by Equation 3, Equation 7 becomes

$$\frac{\cos \omega \cos v - \sin \omega \sin v}{(1 + e \cos v)} = \chi \cdot \frac{di}{dt_{\max}} \quad (8)$$

Solving for v yields the following:

$$v = \cos^{-1}\left(\frac{X}{\sqrt{Y^2 + 1}}\right) \cdot \text{sign}(Y) - \tan^{-1}(Y) \quad (9)$$

and

$$v = \cos^{-1}\left(\frac{X}{-\sqrt{Y^2 + 1}}\right) \cdot \text{sign}(Y) - \tan^{-1}(Y) + 180^\circ \quad (10)$$

where

$$X = \frac{\chi \cdot \frac{di}{dt}_{\max}}{\cos(\omega) - \chi \cdot \frac{di}{dt}_{\max}} \cdot e \quad (11)$$

and

$$Y = \frac{\sin(\omega)}{\cos(\omega) - \chi \cdot \frac{di}{dt}_{\max}} \cdot e \quad (12)$$

Again, since only the magnitude of di/dt is of interest here, Equations 9 and 10 must be solved for both positive and negative values of χ . Two or four solutions will be generated, depending on the threshold value and the eccentricity of the orbit. The boundaries between the thrusting and coasting regions in Figure 6 were determined using this method.

COMPUTATIONAL RESULTS

Performance comparisons between the discontinuous-thrust control law discussed above, and optimal continuous-thrust inclination changes are shown on the following pages. Three cases are considered below, to give the reader some indication of the benefits available with discontinuous-thrust inclination control in various situations.

Optimal continuous-thrust cases are generated using SEPSOT⁴, a low-thrust trajectory optimization program that uses an orbital averaging integration method, and uses calculus of variations to find minimum-time transfers. Discontinuous-thrust control laws were generated via SNAP, a RK78 trajectory propagation tool used under development at NASA Glenn Research Center. The discontinuous-thrust examples below were integrated in Cartesian coordinates. All tests were run neglecting j_2 and higher order gravity effects, SEPSOT and SNAP would both be expending propellant only to achieve the desired inclination change. The spacecraft specifications for the following examples are as follows:

$p = 75 \text{ kW}$
 $\eta_p = 0.70$
 $m_0 = 10000 \text{ kg}$

Example 1: Circular Orbit Inclination Change

In the example below the case of an inclination change of a circular orbit is examined. Since the radius remains constant throughout an orbit, Equation 3 reduces to a simple cosine function.

The purpose of this example is to show whether a discontinuous-thrust inclination change control law can provide any significant propellant mass or trip time benefits over the continuous-thrust case at low eccentricities.

Initial Orbit:	Final Orbit:
$a = 42161 \text{ km}$	$a = 42161 \text{ km}$
$e = 0.000$	$e = 0.000$
$i = 28.45^\circ$	$i = 51.6^\circ$
$\Omega = 0^\circ$	$\Omega = 0^\circ$
$\omega = 0^\circ$	$\omega = 0^\circ$

Table 1: SEPSOT Results from Example 1

I_{sp} (sec)	MET (days)	m_p (kg)
3800	76.1	497
3500	70.0	539
3000	59.7	626

Table 2: Discontinuous-thrust Results for Example 1

χ	3800 sec		3500 sec		3000 sec	
	MET (days)	m_p (kg)	MET (days)	m_p (kg)	MET (days)	m_p (kg)
0.0	78.2	511	71.8	553	61.3	642
0.1	78.7	481	72.3	521	61.7	605
0.2	79.8	456	73.4	494	62.7	574
0.3	82.2	433	75.6	469	64.5	545
0.4	85.6	414	78.7	448	67.2	521
0.5	90.7	395	83.3	429	71.0	499
0.6	98.6	379	90.6	411	77.3	477
0.7	110.3	364	101.2	395	86.2	460
0.8	131.1	351	120.6	381	102.8	443
0.9	182.0	339	167.6	367	144.6	427

As expected, trip time increases and propellant mass expenditure decreases in the discontinuous-thrust cases, shown in Table 2.

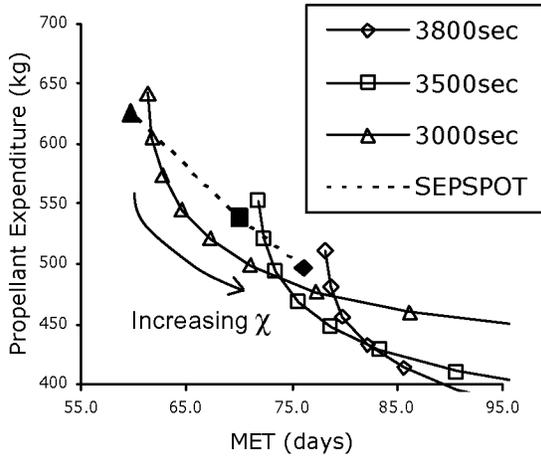


Figure 7: Trip Time vs. Propellant Mass Comparisons for Continuous-Thrust and Discontinuous-Thrust Inclination Changes in a Circular Orbit.

Figure 7, above, shows tradeoff in trip time and propellant expenditure in graphical form. The results of the SEPSPOT continuous-thrust cases (see Table 1) are shown for comparison.

For a given I_{sp} , increasing the value of χ can only reduce propellant mass compared to the continuous-thrust case at the expense of trip time. However, reducing specific impulse and using an appropriate value for χ can achieve savings in both time and propellant consumption.

In this example, $\chi=0.0$ yields results that are inferior to the SEPSPOT results in both trip time and propellant mass. This discrepancy is a result of the fact that SEPSPOT optimizes only continuous-thrust cases. The solutions it finds utilize a portion of the spacecraft's thrust to increase the semi-major axis of the orbit, thereby increasing the magnitude of di/dt throughout the orbit. The relationship between di/dt and semi-major axis can be explained by starting with Equation 1 and the definitions of radius and mean motion⁵. Equation 1 can be rewritten as

$$\frac{di}{dt} = \frac{W\sqrt{1-e^2} \cos(\omega + \nu)}{\sqrt{\frac{\mu}{a}}(1 + e \cos(\nu))} \quad (13)$$

Only the average change in di/dt throughout an orbit is of interest, thus terms involving true-anomaly will be neglected. Also, since di/dt is proportional to the orbit-normal thrust magnitude, W , the term is dropped. The relationship between the average value of di/dt and semi-major axis can then be expressed as

$$\frac{di}{dt}_{avg} \propto \sqrt{\frac{\mu}{a}}(1-e^2) \quad (14)$$

Figure 8 shows the relative value of di/dt , from Equation 14, throughout a SEPSPOT-generated transfer from Example 1.

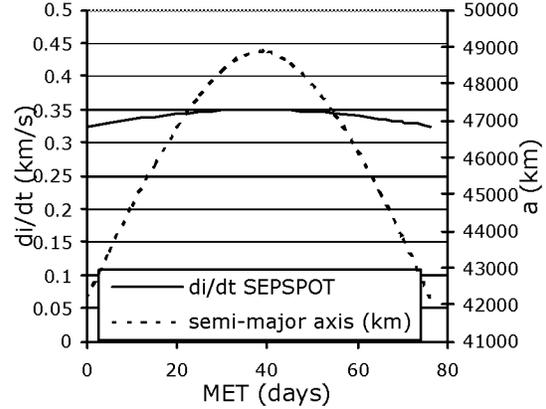


Figure 8: Relative Effect of Semi-Major Axis Change on di/dt

SEPSPOT achieves an increase in di/dt of approximately 10% by increasing the semi-major axis for a portion of the transfer. The discontinuous-thrust cases run here use no in-plane thrust component to modify semi-major axis. Increased performance from the discontinuous-thrust case might be realized by directing thrust in-plane for di/dt inefficient portions of the orbit. However, the combination of the reduction of I_{sp} and suspension of thrust throughout portions of the orbit can more than compensate for the difference in performance, even without changing the semi-major axis.

Example 2: Eccentric Orbit Inclination Change, $\omega=0^\circ$

The second example mission compares continuous and discontinuous-thrust inclination changes in a highly elliptic orbit. The maneuver transfers a spacecraft from an inclination of 28.45° to a final inclination of 63.4° . Other orbital elements are not changed, and the semi-major axis and eccentricity of the orbit are the same as those for a Molniya orbit. Continuous-thrust and discontinuous-thrust results are shown in Tables 3 and 4, respectively.

Initial Orbit:	Final Orbit:
a = 26600 km	a = 26600 km
e = 0.73	e = 0.73
i = 28.45°	i = 63.4°
$\Omega = 0^\circ$	$\Omega = 0^\circ$
$\omega = 0^\circ$	$\omega = 0^\circ$

Table 3: SEPSHOT Results for Example 2

I_{sp} (sec)	MET (days)	m_p (kg)
3800	53.9	352
3500	49.5	382
3000	42.3	444

Table 4: Discontinuous-thrust Results for Example 2

χ	3800 sec		3500 sec		3000 sec	
	MET (days)	m_p (kg)	MET (days)	m_p (kg)	MET (days)	m_p (kg)
0.0	58.0	379	53.3	411	45.6	478
0.1	58.4	352	53.7	382	45.8	443
0.2	59.6	328	54.8	354	46.8	412
0.3	60.7	316	55.8	342	47.7	398
0.4	62.5	304	57.6	330	49.2	384
0.5	65.3	293	60.1	318	51.3	370
0.6	69.7	283	64.2	307	54.8	357
0.7	76.8	274	70.7	297	60.3	345
0.8	90.1	264	82.7	287	70.7	334
0.9	122.6	255	112.6	277	96.2	323

For this case, a discontinuous-thrust inclination control law using only orbit-normal thrust can come close to the performance of SEPSHOT, as Figure 9 shows. Since da/dt is at its maximum value at periapsis and di/dt is at its maximum at apoapsis, SEPSHOT is able to increase the semi-major axis of the orbit and boost the magnitude of di/dt while incurring almost a minimal gravity loss penalty. However, even with purely orbit normal thrust the discontinuous control law can come close to matching the performance of SEPSHOT.

If χ is significantly large such that the spacecraft is not thrusting out-of-plane at periapsis, a tangential thrust component or “perigee-hold¹” steering law can be introduced to modify the semi-major axis without taking any thrusting time away from the orbit-normal component. Greater efficiency can certainly be achieved through the use of a more complex control law that blends both in-plane and orbit-normal thrust in an effective manner.

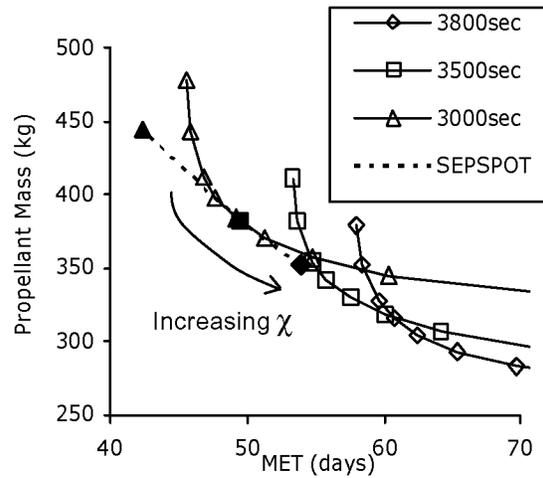


Figure 9: Trip Time vs. Propellant Mass Comparisons for Continuous-Thrust and Discontinuous-Thrust Inclination Changes in a Highly Elliptical Orbit, $\omega=0^\circ$

Example 3: Eccentric Orbit Inclination Change, $\omega=90^\circ$

For thoroughness, the inclination change performed in the previous example is reexamined for the case where $\omega=90^\circ$. All other orbital elements are the same as those in the previous example. The results for continuous-thrust and discontinuous-thrust cases are shown in Tables 5 and 6, respectively.

Table 5: SEPSHOT Results for Example 3

I_{sp} (sec)	MET (days)	m_p (kg)
3800	148.3	969
3500	136.0	1057
3000	115.5	1211

Table 6: Discontinuous-thrust Results for Example 3

χ	3800 sec		3500 sec		3000 sec	
	MET (days)	m_p (kg)	MET (days)	m_p (kg)	MET (days)	m_p (kg)
0.0	144.4	943	132.3	1018	112.1	1175
0.1	145.9	890	133.7	961	113.5	1110
0.2	148.8	842	136.5	912	115.8	1051
0.3	153.5	801	140.8	867	119.6	1002
0.4	160.1	767	147.0	830	125.3	958
0.5	170.0	735	156.5	791	133.5	912
0.6	185.3	704	169.9	763	145.6	879
0.7	210.5	676	192.8	733	164.0	850
0.8	263.5	646	244.9	698	218.9	786
0.9	397.9	571	363.9	615	316.4	737

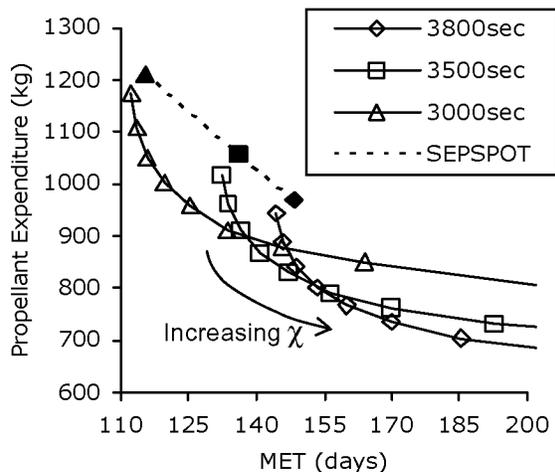


Figure 10: Trip Time vs. Propellant Mass Comparisons for Continuous-Thrust and Discontinuous-Thrust Inclination Changes in a Highly Elliptical Orbit, $\omega=90^\circ$

The results shown in Figure 10 indicate that the discontinuous-thrust control scheme is outperforming SEPSPOT at values of $\chi=0.0$ even at the same specific impulse. It is likely that since the magnitudes of both da/dt and di/dt are small near apoapsis, the fact that SEPSPOT is constrained to using continuous-thrust essentially wastes propellant during the apoapsis dwell. Even greater reductions in propellant and trip time can be achieved by reducing the specific impulse. The discontinuous-thrust control law is clearly preferable to continuous-thrust for inclination changes in orbits where periapsis occurs at or near the antinodes.

CONCLUSIONS

Using the discontinuous-thrust control law for inclination changes can reduce both the propellant load and the trip time required for an inclination-changing maneuver. While the performance benefits of such a method are dependent upon the argument of periapsis, its effectiveness has been demonstrated for both circular and highly elliptical orbits.

While the discontinuous-thrust control law presented here is capable of outperforming continuous-thrust, calculus of variations solutions, like those generated by SEPSPOT, they are at the same time relatively simple. Through the use of this method, the mission designer has a significant amount of control in being able to modify the trip time for an inclination change maneuver to achieve an acceptable propellant expenditure, or vice versa.

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13. ABSTRACT (Maximum 200 words) The evolution of low-thrust propulsion technologies has reached a point where such systems have become an economical option for many space missions. The development of efficient, low trip time control laws has received an increasing amount of attention in recent years, though few studies have examined the subject of inclination changing maneuvers in detail. A method for performing economical inclination changes through the use of an efficiency factor is derived from Lagrange's planetary equations. The efficiency factor can be used to regulate propellant expenditure at the expense of trip time. Such a method can be used for discontinuous-thrust transfers that offer reduced propellant masses and trip-times in comparison to continuous thrust transfers, while utilizing thrusters that operate at a lower specific impulse. Performance comparisons of transfers utilizing this approach with continuous-thrust transfers are generated through trajectory simulation and are presented in this paper.				
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